BRILL

PHRONESIS 60 (2015) 267-309

PHRONESIS

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The Beginnings of Formal Logic: Deduction in Aristotle's *Topics* vs. *Prior Analytics*

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Abstract

It is widely agreed that Aristotle's *Prior Analytics*, but not the *Topics*, marks the beginning of formal logic. There is less agreement as to why this is so. What are the distinctive features in virtue of which Aristotle's discussion of deductions (*syllogismoi*) qualifies as formal logic in the one treatise but not in the other? To answer this question, I argue that in the *Prior Analytics*—unlike in the *Topics*—Aristotle is concerned to make fully explicit all the premisses that are necessary to derive the conclusion in a given deduction.

Keywords

formal logic – deduction – *syllogismos* – premiss – *Prior Analytics* – *Topics*

1 Introduction

It is widely agreed that Aristotle's *Prior Analytics* marks the beginning of formal logic.[1] Aristotle's main concern in this treatise is with deductions (*syllogismoi*). Deductions also play an important role in the *Topics*, which was written before the *Prior Analytics*.[2] The two treatises start from the same definition of what

1 See e.g. Cornford 1935, 264; Russell 1946, 219; Ross 1949, 29; Bocheński 1956, 74; Allen 2001, 13; Ebert and Nortmann 2007, 106-7; Striker 2009, p. xi.

2 While the chronological order of Aristotle's works cannot be determined with any certainty, scholars agree that the *Topics* was written before the *Prior Analytics*; see Brandis 1835, 252-9; Ross 1939, 251-2; Bocheński 1956, 49-51; Kneale and Kneale 1962, 23-4; Brunschwig 1967,

 | DOI 10.1163/15685284-12341286

a deduction is (stated in the first chapter of each). Nevertheless, the *Topics* is generally not regarded as a work of formal logic.[3]

On what grounds, then, does Aristotle's discussion of deductions qualify as formal in the one treatise but not in the other? What are the features in virtue of which the *Prior Analytics*, but not the *Topics*, is a treatise of formal logic? As it turns out, these questions are not easy to answer. This is partly because Aristotle himself does not use the terminology of 'form' and 'formal' in his logical works.[4] Moreover, there is a wide variety of senses in which a given logical theory might or might not be said to be 'formal'.[5] It is not my intention here to enter into a discussion of these various senses. Rather, my aim in this paper is to identify essential features of the discipline that we today call formal logic, and to show that these features are absent from the *Topics* but present in the *Prior Analytics*. I hope that this will contribute to a better understanding both of what is involved in Aristotle's transition from the former to the latter treatise, and of why this transition is a major advance in the origins of formal logic.

I begin with some negative points. First, the syllogistic developed by Aristotle in the *Prior Analytics* is not formal in the sense of being symbolic.[6] Aristotle does not employ a symbolic language containing artificial symbols. Instead, he uses ordinary (if somewhat contrived) Greek, consisting of letters and words familiar to every reader of Greek.

Secondly, the syllogistic is not formal in the sense of being formalized. As Benjamin Morison has shown, Aristotle does not employ a formalized language in the *Prior Analytics*.[7] Whether or not a given argument counts as a deduction (*syllogismos*) in the *Prior Analytics* cannot be judged solely by attending to the linguistic expressions involved without taking into account their meaning. This is because Aristotle does not introduce a canonical way of expressing the premisses and conclusions of his deductions, but allows for a variety of interchangeable expressions such as 'A belongs to all B', 'A is predicated of all B',

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pp. lxxxvi-lxxxix; Corcoran 1974, 88; Barnes 1981, 43-8; Allen 1995, 177-9; 2001, 21; 2007, 88-92; 2011, 65-6; Primavesi 1996, 60; Rapp 2000, 20; Striker 2009, p. xii.

3 Thus, Aristotle's treatment of deductions in the *Topics* is sometimes characterized as 'informal' (Burnyeat 1994, 31) or 'pre-formal' (Woods and Irvine 2004, 34 and 38); see also Allen 2011, 65.

4 See Barnes 1990, 39-40; 2007, 277-82. As Burnyeat (2001, 8) points out, Aristotle's distinction between matter and form is absent from the *Organon*.

5 For an overview of these senses, see MacFarlane 2000, 31-78; Dutilh Novaes 2011.

6 This has been pointed out by Barnes 2007, 274.

7 Morison 2012, 172-3 and 186-7; *pace* Smiley 1982/83, 1.

'B is in A as a whole', and 'A follows all B'. Aristotle does not specify a closed list of canonical expressions to be used in deductions. Any expression is admissible as long as it has the same meaning as the expressions just mentioned.[8] By contrast, the Stoics specified a limited set of canonical expressions in their syllogistic. If one of these canonical expressions is replaced by a non-canonical expression in a deduction, the resulting argument will fail to be a deduction even if the expressions have the same meaning.[9] Thus, the Stoic syllogistic is formalized in that whether or not an argument counts as a deduction can be ascertained from the linguistic expressions alone without attending to their meaning, by determining whether the argument contains a suitable arrangement of canonical expressions. Aristotle's syllogistic, however, is not so formalized.[10] This is what Alexander of Aphrodisias has in mind when he writes that for Aristotle, unlike for the Stoics, 'a deduction has its being not in the words but in what is signified by the words'.[11] Łukasiewicz puts the point succinctly (1957, 15):

> Aristotelian logic is formal without being formalistic, whereas the logic of the Stoics is both formal and formalistic.[12]

If Aristotle's syllogistic is not formalistic or formalized, what is the distinctive sense in which it is formal? One might point out that it is formal in that it is completely general and topic-neutral, applicable to any subject matter whatsoever.[13] This is certainly true. But it is equally true for the *Topics*. The dialectical method presented in the *Topics* and the deductions involved in it are no less general and topic-neutral than the deductions studied in the *Prior*

8 See Brunschwig 1969, 3-5.

9 Frede 1974*a*, 5-6 and 13; Barnes 2007, 314-21.

10 See Łukasiewicz 1957, 15-19; Frede 1974*a*, 13-15; 1974*b*, 198-201; Morison 2012, 186-7.

11 οὐκ ἐν ταῖς λέξεσιν ὁ συλλογισμὸς τὸ εἶναι ἔχει ἀλλ' ἐν τοῖς σημαινομένοις, Alexander, *in APr.* ii.1, 372.29-30 Wallies; similarly, 373.16-17. Alexander describes the Stoic account of deductions as follows: 'They [the Stoics] do not call such arguments deductions since they attend to expression and language (εἰς τὴν φωνὴν καὶ τὴν λέξιν βλέποντες), whereas Aristotle, where the same object is signified, looks to what is signified and not to the expressions.' (*in APr.* ii.1, 84.15-17 Wallies).

12 Similarly, Bocheński 1956, 113 and 124; Mueller 1974, 51.

13 This is one of the senses in which logic is sometimes said to be 'formal' (see e.g. Dutilh Novaes 2011, 314-16).

Analytics.[14] Thus, generality and topic-neutrality do not help us to characterize the relevant difference between the two treatises.

Finally, it is often thought that the specific kind of formality exhibited by the *Prior Analytics* is to be attributed to Aristotle's use in this treatise of schematic letters such as 'A' and 'B' in place of concrete terms such as 'man', 'animal', and 'walking'.[15] For example, Jan Łukasiewicz (1957, 7-8 n. 1), who calls these letters variables, maintains that 'the introduction of variables into logic is one of Aristotle's greatest inventions . . . [B]y using variables Aristotle became the inventor of formal logic'. Similarly, Gisela Striker (2009, p. xii) holds that 'the crucial innovation . . . that makes syllogistic a formal system is the introduction of letters as placeholders for the terms'.

Clearly, Aristotle's schematic letters are a very useful and important piece of notation. Nevertheless, it is doubtful whether they are the crucial feature in virtue of which the *Prior Analytics* marks the beginning of formal logic. Suppose for a moment that Aristotle had used these letters in formulating some of his *topoi* in the *Topics*. For example, consider the following part of a *topos* from the fourth book (4.1, 121a25-6):

> Of those of which the species is predicated, the genus must be predicated as well.

Aristotle could have rewritten this sentence as follows:

> If A is a genus of B, B is a species of A, and B is predicated of C, then A is predicated of C.

If Aristotle had used this alternative notation in the *Topics*, should we then consider the *Topics* to be the beginning of formal logic? The answer, it seems to me, is negative, at least if by formal logic is meant a certain kind of logical theory rather than a form of notation used to express that theory. For the *topoi*

14 Alexander, *in Top.* ii.2, 3.25-4.10 and 5.4-13 Wallies; Smith 1993, 338-9; Code 1999, 45. Aristotle holds that dialectic is 'concerned with things which are, in a way, common for all to know, not for any separate science' (*Rhet.* 1.1, 1354a1-3; similarly *APo.* 1.11, 77a31). Accordingly, Aristotle's account of dialectical deductions in the *Topics* is sometimes called 'formal' on the grounds that its *topoi* are topic-neutral and provide general argument forms applicable to a large number of particular cases (Allen 2001, 15-16 and 69-72; Smith 1997, pp. xxiv and xxvi; Wagner and Rapp 2004, 8; Wagner 2011, 356).

15 For example, Cornford 1935, 264-5; Ross 1949, 29; Łukasiewicz 1957, 7-8 and 13-14; Striker 2009, p. xii.

expressed by the alternative notation would be exactly the same as the ones expressed by Aristotle's original formulation.

Conversely, schematic letters are not essential to Aristotle's project in the *Prior Analytics* either. For Aristotle is able to express his syllogistic moods without schematic letters by way of circumscription. For example, when introducing the moods Barbara and Celarent in *Prior Analytics* 1.4, he first presents them by way of circumscription and only then by means of schematic letters (25b32-9):

> When three terms are so related to one another that the last is in the middle as in a whole and the middle either is or is not in the first as in a whole, it is necessary that there is a syllogism of the extremes ... For if A is predicated of all B and B of all C, it is necessary that A is predicated of all C.

In the first sentence of this passage, Aristotle introduces Barbara and Celarent by way of circumscription; in the last sentence, he presents them schematically. While the circumscription in the first sentence is not fully fleshed out, Aristotle could easily supply the missing details to make it precise. In fact, he could use circumscriptions instead of schemata throughout the entire *Analytics*.[16] The presentation would be lengthier and less perspicuous, but in principle nothing prevents Aristotle from giving a purely circumscriptive presentation of the syllogistic. If he did so, should we then deny the *Prior Analytics* the status of formal logic? Again, the answer, I think, is negative. For Aristotle's theory of syllogistic moods in the three figures would be exactly the same as the one presented by means of schematic letters.[17]

In sum, the distinctive kind of formality exhibited by the *Prior Analytics* but not by the *Topics* cannot be attributed to its use of schematic letters. Nor can it be attributed to its being symbolic, formalized, general or topic-neutral. The purpose of this paper is to give a more adequate account of the kind of formality in question. I begin by examining Aristotle's approach in the two treatises to indeterminate premisses, that is, premisses which lack quantifying expressions such as 'all' and 'some' (Sections 2 and 3). In the *Topics*, such quantifying expressions may be omitted in a deduction as long as they are tacitly understood by

16 See Barnes 2007, 286-92 and 358.

17 This is not to deny that the use of schematic letters is an effective means for Aristotle to present his syllogistic in such a way that it qualifies as formal logic (see n. 48 below). The point is that their use is not essential to achieving this goal since it can be achieved by other means.

the interlocutors. In the *Prior Analytics*, by contrast, they may not be omitted if they are relevant to an argument's counting as a deduction, since Aristotle requires that nothing of relevance be left to implicit understanding between speaker and hearer. In this respect, Aristotle's syllogistic is akin to modern systems of formal logic such as, for example, Gottlob Frege's Begriffsschrift. Furthermore, the syllogistic is akin to these systems in that it aims at gapless deductions in which no premiss is missing (Section 4). Unlike the *Topics*, the *Prior Analytics* provides a criterion for determining when no premiss is missing in a given argument (Section 5). The criterion is based on the assumption that ambiguous terms are excluded from consideration (Section 6). This will allow us to specify four essential features of formal logic that are present in the *Prior Analytics* but not in the *Topics*, such that it is correct to say that the former but not the latter marks the beginning of formal logic (Section 7).

2 Premisses and Problems

In the first chapter of the *Prior Analytics*, Aristotle defines a premiss as a certain kind of *logos* (24a16-17):

> A premiss is a *logos* affirming or denying something of something.

The term *logos* in this definition is often translated as 'sentence'.[18] Thus, premisses are taken to be linguistic expressions rather than non-linguistic items signified by expressions. This is confirmed by Aristotle's discussion in *De Interpretatione* 4 and 5, where affirmations and denials are characterized as significant utterances.[19] Thus, Aristotle takes affirmations and denials, including the premisses introduced in the first chapter of the *Prior Analytics*, to be linguistic expressions endowed with signification.[20]

18 For example, Owen 1889, 80; Smith 1989, 1; Striker 2009, 1; Crivelli 2012, 113.

19 See φωνὴ σημαντική at *Int.* 4, 16b26. Accordingly, the occurrences of λόγος in *Int.* 4 and 5 are usually translated as 'sentence' (e.g. Tredennick in Cooke, Tredennick and Forster 1938, 121; Kneale and Kneale 1962, 45-6; Ackrill 1963, 45-5 and 124-5; Lear 1980, 104; Crivelli 2004, 74). Aristotle's characterization of premisses in *APr.* 1.1 (24a16-17) seems to rely on his discussion in *Int.* 4 and 5 (see Alexander, *in APr.* ii.1, 10.13-12.3 Wallies; Ammonius, *in APr.* iv.6, 15.14-17.10 Wallies; Smith 1989, p. xvii).

20 See Crivelli and Charles 2011, 194; Crivelli 2012, 113-14.

Aristotle proceeds to distinguish three kinds of premisses (*APr.* 1.1, 24a16-22):

> A premiss . . . is either universal or particular or indeterminate (ἀδιόριστος). By universal I mean belonging to all or to none; by particular, belonging to some, or not to some, or not to all; by indeterminate, belonging or not belonging without universality or particularity, as in 'Contraries are studied by the same science' or 'Pleasure is not good'.[21]

This classification concerns the presence or absence of quantifying expressions. Universal premisses contain quantifying expressions such as 'all' and 'no' (as in 'All pleasure is good'). Particular premisses contain expressions such as 'some' and 'not all' (as in 'Some pleasure is good'). Indeterminate premisses differ from universal and particular ones in that they do not contain any quantifying expressions.[22] For example, the sentence 'Pleasure is good' is indeterminate. It is indeterminate even if it is actually true that all pleasure is good. Likewise, 'Contraries are studied by the same science' is indeterminate, even if someone utters it with the intention of asserting that all contraries are studied by the same science. Thus, Aristotle's classification is based on the syntactic criterion of whether or not certain expressions are present in a given sentence.

Now, Aristotle presents a similar classification at the beginning of the second book of the *Topics* (2.1, 108b34-109a1):

> Of problems some are universal, others particular. Universal problems are such as 'All pleasure is good' and 'No pleasure is good'; particular problems are such as 'Some pleasure is good' and 'Some pleasure is not good'.

This passage deals with what Aristotle calls 'problems'. These are theses which are maintained by one of the participants in a dialectical exchange and which are to be refuted by the opponent.[23] Aristotle distinguishes between

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21 This classification is meant to apply not only to premisses of deductions but also to their conclusions.

22 See Ammonius, *in APr.* iv.6, 18.15-38 Wallies; Ross 1949, 289; Łukasiewicz 1957, 4; Smith 1989, 107; Striker 2009, 77.

23 In *Topics* 1.4, problems are characterized as questions of the form 'Is it the case that P, or not?' (101b32-4). In the present passage, Aristotle departs from this characterization, using the term 'problem' instead to refer to the theses that result from an affirmative or negative answer to these questions (see Alexander, *in Top.* ii.2, 129.16-24 Wallies; Slomkowski 1997, 16).

universal and particular problems. Unlike in the *Prior Analytics*, he does not mention a third class of indeterminate problems. Moreover, his classification does not seem to rely on the syntactic criterion that governs the classification in the *Prior Analytics*. Although Aristotle's examples in the passage just quoted contain quantifying expressions such as 'all' and 'some', he does not seem to attach any importance to the presence of such expressions in the rest of the second book of the *Topics*. In fact, he virtually never uses such quantifying expressions after the first chapter of the second book.[24] For example, consider the following passage from *Topics* 2.2 (109b13-24):

> Another *topos* is to examine the items to each or to none of which [a predicate] has been said to belong. Look at them species by species ... For example, if someone has said that opposites are studied by the same science (εἰ τῶν ἀντικειμένων τὴν αὐτὴν ἐπιστήμην ἔφησεν εἶναι),[25] you must examine whether relative opposites, contraries, terms opposed as privation and possession, and contradictory terms are studied by the same science.... For if it is shown in any instance that the science is not the same, we shall have demolished the problem.

This *topos* provides a method for refuting problems in which a predicate is said to belong to *every* (or to *no*) member of a given class of items. In Aristotle's example, the problem is indicated by the indeterminate sentence 'Opposites are studied by the same science'. Although this sentence does not contain a quantifying expression, Aristotle uses it in the present passage to convey the universal claim that *all* opposites are studied by the same science (i.e. that for any given pair of opposites, the two members of the pair are studied by the same science). Likewise, a participant in a dialectical debate might use the sentence to convey this universal claim. The *topos* aims to refute the universal claim by setting out a specific pair of opposites that are not studied by the same science.

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24 See Brunschwig 1967, p. lx.

25 The translation 'Opposites are studied by the same science' reflects the fact that Aristotle takes this sentence to have a predicative structure in which 'opposites' (τῶν ἀντικειμένων) is the subject term and 'studied by the same science' (ἡ αὐτὴ ἐπιστήμη) is the predicate term; see *APr.* 1.36, 48b4-9; 2.26, 69b8-9; Ross 1949, 289; Smith 1989, 107; Barnes 1996, 186 n. 35; Primavesi 1996, 122-3.

Aristotle clearly regards the problems targeted by this *topos* as universal in the sense introduced in 2.1.[26] In 3.6, he refers to this *topos* by means of the phrase 'as in the case of universal problems' (καθάπερ ἐν τοῖς καθόλου προβλήμασιν, 120a33-4).[27] But the reason for regarding the problem in Aristotle's example as universal cannot be the presence of a quantifying expression in the sentence 'Opposites are studied by the same science'. There is no such expression. Rather, the reason seems to be the intended meaning of the sentence as understood by the participants in the dialectical debate. As Gisela Striker points out, most of 'the theses discussed in the *Topics* ... are expressed without explicit quantification, but must be understood as being universal'.[28]

The same is true for premisses in the *Topics*. Universal premisses are frequently indicated by sentences that do not contain quantifying expressions. Examples are 'The angry person desires revenge on account of an apparent slight' and 'Someone who has lost knowledge of something has forgotten it'. Aristotle takes each of these two indeterminate sentences to indicate a universal premiss (καθόλου πρότασις), conveying a universal claim about *every* angry person and about *everyone* who has lost knowledge of something.[29]

On the other hand, indeterminate sentences can also be understood as conveying not a universal but a particular claim. For example, as Aristotle explains in *Topics* 2.4, the indeterminate sentence 'Animal is winged' can be used to convey the particular claim that *some* animals are winged. In this case, 'Animal is winged' is true, whereas 'Man is winged' is false (111a25-9; cf. 111a20-3):[30]

26 In *Topics* 2.1, he distinguishes between universal and particular problems. The latter are discussed in 3.6. This suggests that 2.2-3.5 is intended to deal with universal problems (see Alexander, *in Top.* ii.2, 279.12-17 Wallies; Slomkowski 1997, 134). Whether or not all *topoi* in these chapters deal with universal problems, the present one at 109b13-24 is clearly one of the best candidates (see Brunschwig 1967, p. lx; Primavesi 1996, 112-13).

27 See Pacius 1597*a*, 625; Waitz 1846, 471; Brunschwig 1967, 78; Slomkowski 1997, 153 n. 62.

28 Striker 2009, 194; see also Slomkowski 1997, 24, 27, and 134.

29 The first example is found at *Top*. 8.1, 156a31-2; Aristotle describes it as a καθόλου πρότασις at 156a28 (see also 156a34). The second example is found at *Top*. 8.2, 157b12-13; Aristotle describes it as καθόλου at 157b3 (see also 157b10). For further examples of universal premisses formulated without explicit quantification, see Slomkowski 1997, 23.

30 See Alexander, *in Top*. ii.2, 158.31-160.3 Wallies; Primavesi 1996, 152-4. Similarly, Aristotle writes in the *Categories*: 'If you will call the individual man grammatical it follows that you will call both man and animal [i.e. the species man and the genus animal] grammatical' (*Cat.* 5, 3a4-5; see Perin 2007, 135).

> It is not necessary that everything that belongs to the genus should also belong to the species; for animal is winged and fourfooted, but man is not (ζῷον μὲν γάρ ἐστι πτηνὸν καὶ τετράπουν, ἄνθρωπος δ' οὔ). But everything that belongs to the species must also belong to the genus; for if man is good, then animal also is good (εἰ γάρ ἐστιν ἄνθρωπος σπουδαῖος, καὶ ζῷόν ἐστι σπουδαῖον).[31]

Similarly, 'Animal is not winged' can be used to convey the particular claim that *some* animals are not winged. In this case, 'Animal is winged' and 'Animal is not winged' are both true (*De Interpretatione* 7, 17b30-3).[32]

One and the same indeterminate sentence can on one occasion be used to convey a universal claim and on another occasion to convey a particular claim.[33] In the former case, the problem indicated by the sentence counts as universal in the *Topics*; in the latter case, it does not count as universal but presumably as particular. Thus, the distinction between universal and particular problems introduced in *Topics* 2.1 is not based on the syntactic criterion of containing quantifying expressions. Rather, it is based on semantic, perhaps also pragmatic, criteria pertaining to what a sentence is understood to mean by the interlocutors on a given occasion of use.

This also helps explain why Aristotle does not mention indeterminate problems in *Topics* 2.1 or elsewhere in the *Topics*. Indeterminacy, as defined in *Prior Analytics* 1.1, is based on a syntactic criterion that is not relevant to the *Topics*' semantic classification. Introducing a class of sentences lacking quantifying

31 This is the Greek text printed by most editors (Bekker 1831, Waitz 1846, Forster in Tredennick and Forster 1960, Brunschwig 1967). On the other hand, Strache and Wallies (1923) and Ross (1958) print ἔστιν (with an orthotone accent on the first syllable: ζῷον μὲν γὰρ ἔστι πτηνὸν and εἰ γὰρ ἔστιν ἄνθρωπος σπουδαῖος, καὶ ζῷον ἔστι σπουδαῖον). Primavesi (1996, 153-4) argues that the latter version should be preferred because it marks an existential reading of ἐστί ('There is a good animal') as opposed to a copula reading ('Animal is good'). However, as Kahn (1973, 420-4) has shown, there is no syntactic or semantic difference between the enclitic and accented forms of ἐστί, but their distribution is largely determined by word order. Contrary to what is sometimes thought, the orthotone accent does not mark an existential reading of ἐστί. The literal translation of both ζῷόν ἐστι σπουδαῖον and ζῷον ἔστι σπουδαῖον is 'Animal is good' (see Kneale and Kneale 1962, 37). The same is true for ἔστιν ἐπιστήμη σπουδαία at *Top.* 2.4, 111a21-2, and ἔστι λευκὸς ἄνθρωπος at *Int.* 7, 17b9-10. The literal translations of these sentences are 'Knowledge is good' (Kneale and Kneale 1962, 37) and 'Man is pale' (Ackrill 1963, 129; Whitaker 1996, 83-94; Jones 2010, 35-40; Weidemann 2012, 106 n. 5).

32 See Whitaker 1996, 91-4; Weidemann 2002, 206-7; Jones 2010, 42-5.

33 Kneale and Kneale 1962, 37.

expressions would cut across the *Topics*' twofold classification of problems into universal and particular ones. In the *Topics*, problems indicated by indeterminate sentences may be classified as universal or as particular, depending on their intended meaning on a given occasion. As a result, there is no need for a third class of indeterminate problems.[34]

There is one passage, in *Topics* 3.6, in which Aristotle refers to an 'indeterminate problem' (ἀδιόριστον πρόβλημα, 120a6). In this passage, however, he uses the term 'indeterminate' in a different sense to pick out a certain subclass of particular problems (3.6 as a whole is devoted to particular problems). Specifically, he uses it to pick out those particular problems that do not determine whether or not the corresponding universal problem is true.[35] For example, the particular problem indicated by 'Some pleasure is good' is indeterminate in this sense because it does not determine whether or not the corresponding universal problem indicated by 'All pleasure is good' is true. On the other hand, the particular problem indicated by 'Some pleasure is good and some pleasure is not good' is not indeterminate in this sense, because it excludes the truth of the universal problem (120a20-4). Thus, the indeterminate problems discussed in *Topics* 3.6 are different from the indeterminate premisses introduced in *Prior Analytics* 1.1.[36] The latter and the syntactic criterion by which they are defined

34 This is not to say that necessarily *every* indeterminate sentence, on *every* occasion of use, indicates either a universal or a particular problem (or premiss) in the *Topics*. One might think that indeterminate sentences can sometimes be used to indicate not a universal or particular problem, but an indeterminate one. For example, 'Pleasure is good' might be thought to indicate an indeterminate problem when it is understood by the interlocutors as conveying neither the universal claim that all pleasure is good nor the particular claim that some pleasure is good, but the indeterminate claim that pleasure is good. However, Aristotle never mentions such a semantic kind of indeterminacy in the *Topics*. Nor is it clear whether he would accept it. While he accepts syntactic indeterminacy of linguistic expressions in the *Prior Analytics*, he may have reasons to be sceptical about semantic indeterminacy of intended meanings. But even if he accepted such indeterminate problems, they would be of less importance than indeterminate sentences since most indeterminate sentences would indicate a universal or particular problem on a given occasion of use.

35 See Alexander, *in Top.* ii.2, 288.12-289.31 Wallies; Maier 1900, 79-80 n. 1; Brunschwig 1967, pp. lix-lxi and 163-4; 1968, 16-18. This alternative sense of 'indeterminate' is also found in the *Prior Analytics* (*APr.* 1.4, 26b14-16; 1.5, 27b20-2, 27b28; 1.6, 28b28-30, 29a6; 1.15, 35b11; see Alexander, *in APr.* ii.1, 66.2-18, 67.3-7, 88.6-8, 88.31-3, 105.22-6 Wallies; Waitz 1844, 383; Maier 1896, 162-3; Brunschwig 1969, 13 and 19; Crivelli 2004, 245 n. 21; Striker 2009, 98-9).

36 Aristotle's discussion of indeterminate problems in 3.6 begins as follows: 'On the one hand, if the problem is indeterminate, there is only one way of demolishing it, for example, if someone has said that pleasure is good or is not good (οἷον εἰ ἔφησεν ἡδονὴν ἀγαθὸν εἶναι ἢ

are foreign to the *Topics*. In the next section we will see how these differences affect Aristotle's treatment of deductions in the two works.

3 Indeterminate Premisses in Deductions

In the *Prior Analytics*, Aristotle takes indeterminate sentences to be similar in logical force, perhaps equivalent, to the corresponding particular ones.[37] Thus, he does not endorse any first-figure schemata which have an indeterminate major premiss, just as he does not endorse any first-figure schemata which have a particular major premiss.[38] Accordingly, Aristotle rejects concrete first-figure arguments which have an indeterminate major premiss (*APr.* 1.24, 41b6-11):

μὴ ἀγαθόν), and has added nothing by way of determination.' (120a6-8) Brunschwig argues that τινά should be inserted after ἔφησεν, so that the clause reads 'if someone has said that *some* pleasure is good or is not good' (Brunschwig 1967, 163-4; 1968, 16-18; followed by Crivelli 2004, 245 n. 21). According to Brunschwig, the insertion is called for because it makes clear that the passage is concerned with the alternative sense of indeterminacy just described. However, even if the passage involves the syntactically indeterminate sentence 'Pleasure is good', 'indeterminate' at 120a6 may still be used in the alternative sense. For the problem indicated by 'Pleasure is good' may be indeterminate in the alternative sense just as the problem indicated by 'Some pleasure is good' (Pacius 1597*b*, 392; Maier 1900, 79-80 n. 1). There is strong independent evidence that 'indeterminate' at 120a6 should be understood in the alternative sense. First, Aristotle's examples at 120a8-20 are indeterminate in the alternative sense but not in the syntactic sense. Moreover, the phrase 'On the one hand, if the problem is indeterminate' (ἀδιορίστου μὲν οὖν ὄντος τοῦ προβλήματος) at 120a6 corresponds to the phrase 'On the other hand, if the thesis is determinate' (διωρισμένης δὲ τῆς θέσεως οὔσης) at 120a20-1. The latter phrase clearly does not pick out the class of sentences that fail to be indeterminate in the syntactic sense defined in *Prior Analytics* 1.1; for Aristotle makes it clear that this phrase does not apply to sentences such as 'Some pleasure is good', but picks out the class of problems that fail to be indeterminate in the alternative sense (see 120a21-31). Thus, it is natural to take the corresponding phrase at 120a6 to pick out the class of problems that are indeterminate in the alternative sense.

37 It is generally agreed that Aristotle takes indeterminate sentences to be equivalent to particular ones in the *Prior Analytics*; see Alexander, *in APr.* ii.1, 30.29-31, 49.15, 62.24, 111.30-112.2, 267.2 Wallies; Philoponus, *in APr.* xiii.2, 79.4-5, 252.35 Wallies; *in Post. An.* xiii.3, 296.10-11 Wallies; Waitz 1844, 369; Bocheński 1951, 43; Kneale and Kneale 1962, 55; Ackrill 1963, 129; Thom 1981, 19; Barnes 1990, 87; 2007, 141. On the other hand, Aristotle does not explicitly assert this equivalence (Barnes 2002, 107), so that there is room for scepticism (see Whitaker 1996, 86-7).

38 *APr.* 1.4, 26a30-9 and 26b21-5; see Alexander, *in APr.* ii.1, 51.30-1 Wallies.

> Moreover, in every deduction one of the terms must be affirmative, and universality must be present ... For let it be proposed to show that musical pleasure is good. Then, if someone should claim that pleasure is good without adding 'all', there will not be a deduction.[39]

In this passage, Aristotle considers someone who wants to deduce the conclusion 'Musical pleasure is good'. She states the premiss 'Pleasure is good' without adding a universally quantifying expression such as 'all' (μὴ προσθεὶς τὸ πᾶσαν). Thus, she puts forward the following argument in the first figure:

A1	Pleasure is good.	(major premiss)
	Musical pleasure is pleasure.	(minor premiss)
	Therefore, musical pleasure is good.	(conclusion)

Aristotle contends that this argument fails to be a deduction because the major premiss is indeterminate (i.e. because it lacks a quantifying expression).[40] The implication is that if this premiss is replaced by the universal affirmative sentence 'All pleasure is good', the resulting argument will be a deduction.[41]

Given that A1 does not count as a deduction in the *Prior Analytics*, A2 will not count as a deduction either because its major premiss is indeterminate:

39 ἔτι τε ἐν ἅπαντι δεῖ κατηγορικόν τινα τῶν ὅρων εἶναι καὶ τὸ καθόλου ὑπάρχειν ... κείσθω γὰρ τὴν μουσικὴν ἡδονὴν εἶναι σπουδαίαν. εἰ μὲν οὖν ἀξιώσειεν ἡδονὴν εἶναι σπουδαίαν μὴ προσθεὶς τὸ πᾶσαν, οὐκ ἔσται συλλογισμός. I follow Tredennick (in Cooke, Tredennick and Forster 1938, 323) in translating τὸ καθόλου in this passage as 'universality' (just as ἄνευ τοῦ καθόλου at 41b7 and at 1.1, 24a20, is translated as 'without universality'; cf. n. 40 below).

40 See Alexander, *in APr.* ii.1, 266.32-267.5 Wallies; Philoponus, *in APr.* xiii.2, 252.31-5 Wallies; Pacius 1597*b*, 155-6; Waitz 1844, 434; Mendell 1998, 185; Striker 2009, 179. It seems clear that Aristotle regards the major premiss of A1 as indeterminate in the sense defined in *Prior Analytics* 1.1 (24a19-22). Aristotle describes this major premiss as being 'without universality' (ἄνευ τοῦ καθόλου, 41b7); the same phrase (ἄνευ τοῦ καθόλου) is used in 1.1 to characterize indeterminate premisses (24a20). Also, the major premiss of A1 is very similar to one of Aristotle's examples of indeterminate premisses in 1.1, 'Pleasure is not good' (24a21-2).

41 The resulting argument will have a universal affirmative major premiss, and an indeterminate affirmative minor premiss and conclusion. Aristotle accepts such arguments as deductions: given that indeterminate sentences are equivalent to the corresponding particular ones (n. 37 above), their validity follows from the validity of Darii. Aristotle can be taken to assert the validity of such arguments at *APr.* 1.4, 26a28-30, and 1.7, 29a27-9; see Alexander, *in APr.* ii.1, 51.24-30 Wallies.

A2	Opposites are studied by the same science.	(major premiss)
	Contraries are opposites.	(minor premiss)
	Therefore, contraries are studied by the same science.	(conclusion)

In the *Topics*, by contrast, Aristotle accepts A2 as a deduction (8.1, 155b30-4):

> If you wish to secure an admission that contraries are studied by the same science, you should make the claim not for contraries but for opposites; for, if this is granted, it will then be deduced (συλλογιεῖται) that contraries are studied by the same science, since contraries are opposites.

In this passage, Aristotle states that the conclusion of A2 is deduced (συλλογιεῖται) from the two premisses. He goes on to describe A2 as a case of establishing a conclusion 'through a deduction' (διὰ συλλογισμοῦ and συλλογισμῷ, 155b35 and 37). Thus he regards A2 as a deduction even though the major premiss lacks a quantifying expression.[42] Presumably, he does so on the grounds that the major premiss is understood as conveying the universal claim that all opposites are studied by the same science.

It is instructive to compare this with Aristotle's discussion of indeterminate sentences in *De Interpretatione* 7. There, Aristotle calls them sentences 'stating of a universal not universally' (17b6-12). They state something 'of a universal' because, in the *De Interpretatione*, Aristotle requires that their subject be a general term such as 'man' as opposed to a singular term such as 'Callias'. They state it 'not universally' because they lack quantifying expressions. Aristotle contrasts them with universal sentences, which contain universally quantifying expressions. Universal affirmative sentences, such as 'Every man is pale', are contrary to the corresponding universal negative ones, such as 'No man is pale' (17b3-6).[43] This is not the case for indeterminate sentences (17b6-10):

> But when one states something of a universal but not universally, the sentences are not contrary, though what is being conveyed may be contrary (τὰ μέντοι δηλούμενα ἔστιν εἶναι ἐναντία); examples of what I mean by 'stating of a universal not universally' are 'Man is pale' and 'Man is not pale'.

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42 See Slomkowski 1997, 26-7.

43 'Every man is pale' and 'No man is pale' are ἐναντίαι ἀποφάνσεις (17b4-5). In the *De Interpretatione*, ἀποφάνσεις are taken to be λόγοι, that is, sentences (see Kneale and Kneale 1962, 45-6; Ackrill 1963, 124-5; Lear 1980, 104; Crivelli 2004, 86-7; cf. n. 19 above). Thus, Aristotle holds that the sentence 'Every man is pale' is contrary to the sentence 'No man is pale'.

In this passage, Aristotle states that an indeterminate affirmative sentence such as 'Man is pale' is not contrary to the corresponding indeterminate negative sentence 'Man is not pale'. He adds, however, that 'what is being conveyed may be contrary'. By this he seems to mean that an indeterminate sentence 'may on occasion be intended universally' (Ackrill 1963, 129). A speaker may use 'Man is pale' and 'Man is not pale' to convey the same thing she would convey by using the corresponding universal sentences 'Every man is pale' and 'No man is pale', respectively. When a pair of indeterminate sentences is used in this way, what is conveyed by them (τὰ δηλούμενα) is contrary.[44] Still, the sentences themselves, unlike the universal sentences, are not contrary. After all, as we have seen, indeterminate sentences may also be used to convey the same thing as particular ones. When used in this latter way, 'Man is pale' and 'Man is not pale' are both true (17b29-34). As Aristotle points out (17b34-7), saying that both of these sentences are true . . .

> might seem absurd at first sight, because 'Man is not pale' looks as if it signifies (σημαίνειν) also at the same time that no man is pale. This, however, does not signify the same (οὔτε ταὐτὸν σημαίνει), nor does it necessarily hold at the same time.

According to this passage, it is a mistake to think that 'Man is not pale' signifies the same thing as 'No man is pale'. The two sentences do not signify (σημαίνειν) the same, even when a speaker uses them to convey the same thing. What *the sentences* mean considered in themselves is not the same; what *a speaker* means by uttering them on a given occasion may be the same.[45] In other words, their literal meaning is not the same; but their intended meaning, or speaker meaning, on a given occasion of use may be the same.

In the *Topics*, Aristotle regards A2 as a deduction on the grounds that the indeterminate sentence indicating its major premiss, 'Opposites are studied by the same science', is used to convey a universal claim. In doing so, he attends to the speaker meaning of the sentence rather than to its literal meaning. Now, the *Topics* is a treatise on dialectic. Dialectic, for Aristotle, is 'directed at someone else' (πρὸς ἕτερον; *Top.* 8.1, 155b10 and 27). Thus, dialectical activity essentially consists in argumentation between two interlocutors opposing one another.[46] In such a setting, what is important is not so much the literal meaning of the sentences uttered by the interlocutors but their speaker

44 See Jones 2010, 42-5; cf. Whitaker 1996, 84-5.

45 See Jones 2010, 42-5.

46 See Bolton 1994, 102-3; Primavesi 1996, 35-7.

meaning. Speaker meaning often goes beyond literal meaning and is, at least in part, determined by the speaker's intentions. Given that speaker and hearer are interacting cooperatively in their communication, the hearer will typically be able to understand the intended speaker meaning.[47] For example, the hearer will understand that the indeterminate sentence 'Dogs are mammals' is intended to convey a universal claim, whereas the indeterminate sentence 'Dogs are running down the street' is intended to convey a particular claim.

In interpreting each other's utterances, both participants in a dialectical exchange tacitly rely on a body of shared knowledge and contextual information, along with general pragmatic principles governing cooperative communication. As a result, the precise linguistic formulation of premisses and conclusions, the presence or absence of certain expressions in them, is not essential in the *Topics*. For example, in order for A2 to count as a deduction, it is important that the major premiss be understood as conveying a universal claim, but the universality need not be directly expressed in the sentence by a quantifying expression. As long as it is tacitly understood by the interlocutors, the major premiss counts as universal and A2 counts as a deduction in virtue of the intended speaker meaning.

In the *Prior Analytics*, by contrast, Aristotle does not take into account the speaker meaning of the expressions occurring in a deduction but only their literal meaning. Moreover, he does not take into account the literal meaning of all of these expressions. By using schematic letters in place of concrete subject and predicate terms, Aristotle makes it clear that the specific meaning of these terms is not relevant to an argument's counting as a deduction.[48] It is only the meaning of some expressions that is important, e.g. expressions such

47 As Aristotle explains in *Topics* 8.11 (161a37-b5), dialectical argumentation, unlike eristic or agonistic argumentation, is a cooperative enterprise (see κοινόν ἔργον at 161a37-8; cf. Owen 1968, 106-8). Aristotle writes (161b2-5): 'The person who questions eristically is a poor dialectician (φαύλως διαλέγεται), and so is the answerer who will not grant what is evident or will not understand what it is that the questioner intends to get (μηδ' ἐκδεχόμενος ὅ τί ποτε βούλεται ὁ ἐρωτῶν πυθέσθαι).' According to this passage, someone who refuses to understand the opponent's utterances in the way they are obviously intended is a poor dialectician (Smith 1997, 140; cf. also *Top.* 1.18, 108a24-6). If the opponent refuses to cooperate and behaves peevishly (δυσκολαίνει), the argumentation is not dialectical but agonistic (*Top.* 8.11, 161a23-4). Agonistic and eristic arguments are discussed not in the *Topics* but in the *Sophistici Elenchi* (see *SE* 2, 165b8-11).

48 See Alexander, *in APr.* ii.1, 53.28-54.2, 379.14-380.27 Wallies; Philoponus, *in APr.* xiii.2, 46.25-47.9 Wallies. Moreover, the use of schematic letters helps Aristotle to abstract from speaker meaning since the speaker meaning of a sentence on a given occasion of use will typically depend on the literal meaning of its subject and predicate terms.

as 'is predicated of all' and 'belongs to no'.[49] Thus, if an argument's being a deduction depends on the meaning of a quantifying expression such as 'all', the expression cannot be omitted. Aristotle is not formalistic and does not specify exactly *which* expression to use. But he does require that *some* suitable quantifying expression be present; otherwise the argument will not count as a deduction, even if the quantification is tacitly understood by the interlocutors.

This is confirmed by the fact that Aristotle is concerned not to omit any quantifying expressions in the assertoric syllogistic (*Prior Analytics* 1.1-2 and 4-7). There are only two occurrences of the verb 'belong' (ὑπάρχειν) not accompanied by a quantifying expression in the assertoric syllogistic. Both of them occur in a special kind of context (namely, in proofs by ecthesis) which does not require the presence of a quantifying expression.[50] Thus, Aristotle is very careful about the formulation of premisses and conclusions in the assertoric syllogistic, making sure that all quantifying expression are made explicit.[51]

This degree of precision is significant because Aristotle is generally quite willing to use allusive and elliptical prose. Aristotle thinks that 'we must not

49 In the opening sentence of the *Prior Analytics*, Aristotle promises to elucidate the meaning of 'is predicated of all' and 'is predicated of none' (τί λέγομεν τὸ κατὰ παντὸς ἢ μηδενὸς κατηγορεῖσθαι, *APr.* 1.1, 24a14-15). This elucidation, which is known as the *dictum de omni et de nullo*, is given at the end of the first chapter (24b28-30). Aristotle appeals to this *dictum* in 1.4 to justify the validity of his perfect first-figure schemata (25b39-40, 26a24, 26a27; see Alexander, *in APr.* ii.1, 54.9-11, 55.1-3, 61.3-5, 69.14-20 Wallies; Smith 1989, 111; Byrne 1997, 45-6; Ebert and Nortmann 2007, 292 and 302; Barnes 2007, 392-4; Striker 2009, 83-4). These schemata are used in chapters 1.5-6 to establish the validity of second- and third-figure schemata. Thus, the validity of all these schemata depends on the meaning of expressions such as 'is predicated of all' and 'is predicated of none'.

50 The two occurrences are at 1.6, 28a25 and 28b21; see Smith 1982, 119. For varying accounts of why quantifying expressions are not required in these two passages, see Alexander, *in APr.* ii.1, 100.9-14 Wallies; Smith 1982, 119-20; Malink 2008, 524-30.

51 It should be noted that this is not true for the modal syllogistic (*APr.* 1.3 and 8-22). There, Aristotle occasionally omits quantifying expressions where they should be used (e.g. 1.10, 30b11, 34b3-4; 1.15, 34b23; 1.19, 38b9-10; 1.22, 40a27; cf. also 1.9, 30a18-20 and 1.15, 34a19-21). Once Aristotle has indicated the exact placement of quantifying expressions in the assertoric syllogistic, he takes a more flexible approach in the modal syllogistic (where his focus is on modal rather than quantifying expressions). There is good reason to think that the modal syllogistic is a later insertion added by Aristotle after most of the first book of the *Prior Analytics* was completed (Bocheński 1956, 50-1; Łukasiewicz 1957, 131 n. 1; Corcoran 1974, 88 and 120; Ebert and Nortmann 2007, 110; Striker 2009, 108). If this is correct, then Aristotle's statement in 1.24 that quantifying expressions must not be omitted (41b6-11) originally came just after the assertoric syllogistic (in which Aristotle does not omit any quantifying expressions).

seek the same degree of exactness (ἀκρίβειαν) in all areas, but only such as fit the subject matter and is proper to the investigation (τῇ μεθόδῳ)' (*NE* 1.7, 1098a26-9). Being more precise than is required in a given context is a waste of time (ἀργόν, *Pol.* 7.12, 1331b18-19).[52] For example, when Aristotle introduces various kinds of deduction in the first chapter of the *Topics*, he does not deem it necessary to give an exact account of them because such an account is not required for the kind of investigation pursued in the *Topics* (1.1, 101a18-24):

> Let the above then be a description in outline of the different kinds of deduction (εἴδη τῶν συλλογισμῶν)... It is not our intention to give the exact account (τὸν ἀκριβῆ λόγον) of any of them. What we want to do instead is to describe them in outline, since we deem it fully sufficient, for the purposes of the present investigation (μέθοδον), to be able to recognize each of them in some way or other.

The *Prior Analytics*, by contrast, is intended to provide an exact account of deduction.[53] Presumably Aristotle would not focus on the exact placement of quantifying expressions in this treatise if he did not deem it necessary. Aristotle does not explain why it is necessary; but a natural explanation is that he thought that, for the purposes of the investigation undertaken in the *Prior Analytics*, everything in the premisses and conclusion that is relevant to an argument's counting as a deduction needs to be made explicit by some linguistic expression so that nothing of relevance is left to tacit understanding between speaker and hearer.

If this is Aristotle's rationale, it is in line with basic commitments of modern systems of formal logic that have been developed since the end of the 19th century. For example, Gottlob Frege describes the system introduced in his *Begriffsschrift* as follows (1879, §3):

> All those peculiarities of ordinary language that result only from the interaction of speaker and hearer—as when, for example, the speaker takes the expectations of the hearer into account and seeks to put them on the right track even before the sentence is uttered—have nothing that answers to them in my formula language, since in a judgment I consider

52 See Anagnostopoulos 1994, 131-40. When some degree of inexactness is appropriate for the purposes of a given investigation, Aristotle tends 'to view exactness as something toilsome and as something that reflects the kind of pettiness or meanness that he elsewhere associates with the behavior of the illiberal person' (Anagnostopoulos 1994, 126).

53 See Alexander, *in Top.* ii.2, 26.13-19 Wallies; similarly, Smith 1997, 51.

> only that which influences its *possible consequences*. Everything necessary for a correct inference is expressed in full ...; *nothing is left to guesswork*.[54]

This description applies to any system of formal logic put forward since the Begriffsschrift. If I am correct, it applies equally to Aristotle's syllogistic in the *Prior Analytics* (except that the latter should not be described as a 'formula language' since it is neither symbolic nor formalistic). To be clear, Aristotle's syllogistic differs fundamentally from modern systems of formal logic and should not be assimilated to them. For example, it differs from them in logical syntax, expressive power and in the kinds of inference it approves of.[55] Despite these differences, however, they all have a common objective that often goes unnoticed: to abstract from speaker meaning and not to 'leave anything to guesswork' in their respective accounts of deductive inference. As we shall see in the next section, they share another objective in that they aim at gapless deductions in which no premiss is missing.

4 Missing Premisses

In the first chapter of the *Topics*, Aristotle defines a deduction as a kind of argument in which a conclusion follows necessarily from given premisses 'through' these premisses (100a25-7):

> A deduction is an argument in which, certain things having been supposed, something different from the things supposed results of necessity through the things supposed (διὰ τῶν κειμένων).

54 Similarly, Frege (1979, 213) writes: 'The sentences of our everyday language leave a good deal to guesswork. It is the surrounding circumstances that enable us to make the right guess. The sentence I utter does not always contain everything that is necessary ... But a language that is intended for scientific employment must not leave anything to guesswork.'

55 In fact, Frege regards it as a distinctive achievement of his Begriffsschrift that it departs further from Aristotle's logical syntax than its predecessors (Frege 1979, 15; cf. 1879, p. vii and 2-4); for a discussion of these differences, see Barnes 1996, 175-81 and 196-7; 2007, 93-113. Accordingly, it is sometimes thought that 'Post-Aristotelian logic begins only with Frege' (Sluga 1980, 65), and that 'not until the twentieth century was [Aristotelian logic] finally supplanted as a result of the work of Frege and his successors' (Smith 1995, 27; similarly, Beaney 1996, 37).

This definition is repeated almost verbatim in the first chapter of the *Prior Analytics* (24b18-21).[56] There, the phrase 'through the things supposed' is replaced by the phrases 'in virtue of these things being so' (τῷ ταῦτα εἶναι) and 'because of these things' (διὰ ταῦτα). Aristotle uses these three phrases interchangeably and seems to regard them as equivalent.[57] Thus, the definition of deduction in the *Prior Analytics* is the same as the one in the *Topics*.[58] In both treatises, the definition includes the condition that the conclusion follow through (or because of, or in virtue of) the premisses. Call this the 'causal condition'.

Nevertheless, there are differences in how Aristotle uses the causal condition in the two treatises. In the *Topics*, the most striking use he makes of the condition is to exclude arguments that contain superfluous premisses which are not necessary to deduce the conclusion (8.11, 161b28-30).[59] In the *Prior Analytics*, by contrast, he invokes it to exclude arguments in which premisses are missing. In the first chapter of the *Prior Analytics*, Aristotle elucidates the causal condition by stating that it is tantamount to the condition of 'needing no further term from outside in order for the necessity to come about' (24b20-2). Although Aristotle speaks of terms that are missing, it is generally agreed that he has in mind missing premisses.[60] His point is that all premisses necessary to deduce the conclusion in a given argument must be explicitly stated.[61] No premiss must be missing.

56 Similar definitions are given in *SE* 1, 164b27-165a2 and *Rhet.* 1.2, 1356b16-18.

57 See Barnes 1981, 23-4 n. 9; Rapp 2002, 63 and 164. In *Topics* 8.11 (161b28-30), Aristotle takes the condition expressed by τῷ ταῦτα εἶναι to be part of the definition of deduction (cf. *Rhet.* 1.2, 1356b17; *SE* 6, 168b24). This suggests that he takes this phrase to be equivalent to the one used in *Topics* 1.1 (διὰ τῶν κειμένων).

58 See Barnes 1981, 23-4 n. 9; Bolton 1994, 116; Primavesi 1996, 59 n. 2; Rapp 2000, 17-20; similarly, Striker 2009, 79-81.

59 See Alexander, *in Top.* ii.2, 13.28-14.2 and 568.18-23 Wallies; Frede 1974*a*, 22; Barnes 1980, 168-9; Allen 1995, 189; Smith 1997, 143; Mignucci 2002, 251; Ebert and Nortmann 2007, 227-8; Striker 2009, 81-2.

60 Frede 1974*a*, 22; Ebert and Nortmann 2007, 227; Striker 2009, 81; Keyt 2009, 36; Di Lascio 2014, 274. Although Aristotle does not expressly invoke the causal condition to exclude arguments with superfluous premisses in the *Prior Analytics*, there is reason to think that he still takes it to exclude such arguments; see *APr.* 1.32, 47a14-20, and Frede 1974*a*: 22.

61 Thus, Frede (1974*a*: 22) takes the causal condition to require that 'all the assumptions on which the inference is based have been made explicit'. Similarly, Keyt (2009, 36) takes it to require that 'the conclusion of a syllogism must follow from explicit, rather than tacit or suppressed, premisses'; see also Mignucci 2002, 250-2; Striker 2009, 81. However, this does not mean that absolutely *everything* that is relevant to an argument's counting as a deduction is made explicit in the premisses. For example, the *dictum de omni et de nullo*

While the requirement that no premiss be missing in a deduction may seem obvious, it raises the substantive question of how to determine whether or not a premiss is missing in a given argument in order for it to count as a deduction (*syllogismos*). Aristotle addresses this question in *Prior Analytics* 1.32. As he points out, it can sometimes be difficult to discern whether a premiss is missing. It is especially difficult in arguments in which the conclusion follows necessarily from the premisses even though one or more premisses are missing (47a22-35).[62] Aristotle gives the following example of such an argument (47a24-8):

A3 A substance is not destroyed by the destruction of what is not a substance.
If the things out of which something is composed are destroyed, then what consists of them must also perish.
Therefore, any part of a substance is a substance.

He comments on this argument as follows (47a26-8):

> When these [i.e. the two premisses of A3] have been assumed, it is necessary that any part of a substance be a substance; yet it has not been deduced through the things assumed, but premisses are missing (οὐ μὴν συλλελόγισται διὰ τῶν εἰλημμένων, ἀλλ᾽ ἐλλείπουσι[63] προτάσεις).

Although the conclusion follows necessarily from the premisses, A3 fails to be a deduction because one or more premisses are missing in it (47a31-5). Since it is

is in many cases relevant to an argument's counting as a deduction (n. 49 above), but is not mentioned in the premisses (see Section 5 below). Also, it is relevant to an argument's counting as a deduction that it satisfies the definition of deduction; but Aristotle is clear that this definition should not be mentioned in the premisses of a deduction (*APo.* 2.6, 92a11-19; see Philoponus, *in APo.* xiii.3, 355.10-356.6 Wallies; Barnes 1994, 212-13; Fait 2013, 262-3).

62 For such arguments, see Alexander, *in APr.* ii.1, 21.28-30, 344.9-345.12, 346.27-8 Wallies; Philoponus, *in APr.* xiii.2, 320.16-322.18 and 323.18-27 Wallies; Frede 1974*a*, 20-3; Mignucci 2002, 248-56.

63 I take this occurrence of ἐλλείπειν to mean 'be missing' (following Pacius 1597*a*, 262; Mueller 2006, 30; Ebert and Nortmann 2007, 78; Striker 2009, 52). For this meaning of ἐλλείπειν, see Bonitz, *Index Arist.* 238b5-11. Similarly, the phrase ἐλλείπουσι προτάσεις at 47a28 is sometimes translated as 'premisses have been left out' (von Kirchmann 1877, 75; Smith 1989, 51). This is supported by the parallel phrase 'one of the necessary premisses has been left out' (τι τῶν ἀναγκαίων παραλέλειπται) at 47a19.

not a deduction, A3 must violate one of the conditions laid down in Aristotle's definition of deduction. Presumably, it violates the causal condition.[64]

Aristotle does not specify which premiss or premisses are missing in A3. Alexander suggests that it is a premiss such as 'A whole is composed of its parts'.[65] In any case, whichever premiss or premisses are missing, Aristotle makes it is clear that in order to turn A3 into a deduction these premisses must be added to the argument (47a14-20):

> Sometimes people propose a universal premiss but do not assume the premiss that is contained in it, either in writing or in speech (οὔτε γράφοντες οὔτ' ἐρωτῶντες).[66] Or they ... instead ask for other useless things. So we must see whether something superfluous has been assumed and whether one of the necessary premisses has been left out (τι τῶν ἀναγκαίων παραλέλειπται); and the one should be posited and the other taken away.

When people put forward arguments, they sometimes omit premisses that are necessary to deduce the conclusion. The premisses may be tacitly understood by the interlocutors, but they are not made explicit 'either in writing or in speech'. Such arguments violate the causal condition. For the causal condition, as explained in the first chapter of the *Prior Analytics*, requires that all necessary premisses be expressly stated (see n. 61 above).

When Aristotle says that people sometimes fail to make premisses explicit 'in speech' (ἐρωτῶντες), he seems to be referring to dialectical debates between two interlocutors. It is less clear what he has in mind when he says that they sometimes fail to do so 'in writing' (γράφοντες). Pacius (1597*b*, 185) suggests that Aristotle is referring to mathematical texts. As Striker (2009, 214) points out, this is supported by Aristotle's discussion of a mathematical example in *Prior Analytics* 1.24. In this chapter, as we have seen, Aristotle argues that every deduction must contain a universal premiss. He first illustrates this claim by A1, which fails to be a deduction because the major premiss is not universal

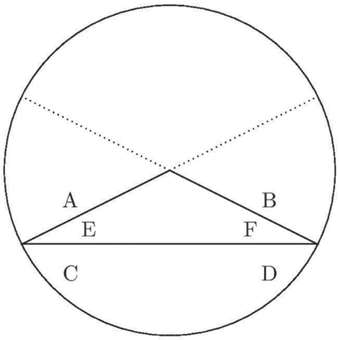
64 Mignucci 2002, 254-6. The causal condition requires that the conclusion follow 'through the things supposed' (διὰ τῶν κειμένων, *Top.* 1.1, 100a26-7; *SE* 1, 165a2), whereas Aristotle denies that the conclusion of A3 has been deduced 'through the things assumed' (διὰ τῶν εἰλημμένων, *APr.* 1.32, 47a27-8).

65 Alexander, *in APr.* ii.1, 347.5-7 Wallies. For alternative suggestions, see Pacius 1597*a*, 262; Ebert and Nortmann 2007, 800-5; Striker 2009, 214. Some of these commentators argue that in order to turn A3 into a deduction one must not only add a premiss, but also transform the two premisses already present in A3.

66 I follow Smith (1989, 51) and Ebert and Nortmann (2007, 78) in rendering ἐρωτῶντες as 'in speech'.

but indeterminate (41b6-11). He then proceeds to illustrate the claim by a series of mathematical arguments which will not count as deductions unless they contain an explicit universal premiss (41b13-22):

> This becomes more evident in geometrical proofs, for example, the proof that the base angles of an isosceles triangle are equal. Let the lines A and B be drawn to the center. Then, if one should assume that angle AC is equal to angle BD without asserting generally that the angles of semicircles are equal, and again if one should assume that C is equal to D without making the additional assumption that this holds for every angle of a segment, and if one should then, lastly, assume that, the whole angles being equal and the angles subtracted being equal, the remaining angles E and F are equal—he will beg the question [and hence fail to give a deduction],[67] unless he assumes [generally] that equal things remain when equals are subtracted from equals.



In this passage, Aristotle discusses a proof to the effect that the base angles of an isosceles triangle, E and F, are equal. Both angles are parts of the larger angles AC and BD, respectively. These are 'mixed' angles formed by the lines A

67 Aristotle does not explain why failure to assume the appropriate universal premiss will result in begging the question (*petitio principii*). Nevertheless, it is clear that such arguments fail to be deductions (Philoponus, *in APr.* xiii.2, 254.12-23 Wallies). In Aristotle's view, an argument that begs the question is not a deduction since his definition of deduction requires that the conclusion be distinct from any of the premisses (ἕτερόν τι τῶν κειμένων, *APr.* 1.1, 24b19; *Top.* 1.1, 100a25-6); see *SE* 5, 167a25-6; 6, 168b25-6; Alexander, *in APr.* ii.1, 18.12-20.29 Wallies; Ammonius, *in APr.* iv.6, 27.34-28.20 Wallies; Frede 1974*a*, 20-1; Bolton 1994, 110-12; Barnes 2007, 487-90; Striker 2009, 80; Crivelli 2012, 125; Di Lascio 2014, 274.

and B and the curve of the circle. The base angles result from subtracting the mixed angles C and D from the larger mixed angles. Thus, Aristotle infers the equality of the base angles from the equality of AC and BD and the equality of C and D.[68] He does so by means of three deductions which can be represented as follows:[69]

A4 All angles of semicircles are equal.
AC and BD are angles of semicircles.
Therefore, AC and BD are equal.

A5 All angles of the same segment of a circle are equal.
C and D are angles of the same segment of a circle.
Therefore, C and D are equal.

A6 All remainders of subtracting equals from equals are equal.
E and F are remainders of subtracting equals from equals.
Therefore, E and F are equal.

Aristotle takes these deductions to be instances of a valid schema in the first figure.[70] Each of them has a universal affirmative major premiss. If this premiss is omitted, the resulting arguments will not be deductions.[71] Thus, Aristotle denies that A7-9 are deductions:

A7 AC and BD are angles of semicircles.
Therefore, AC and BD are equal.

A8 C and D are angles of the same segment of a circle.
Therefore, C and D are equal.

A9 E and F are remainders of subtracting equals from equals.
Therefore, E and F are equal.

68 For the details of this proof, see Alexander, *in APr.* ii.1, 268.6-269.15 Wallies; Philoponus, *in APr.* xiii.2, 253.28-254.12 Wallies; Ross 1949, 374-6. While mixed angles play only a marginal role in Euclid, they were probably more prominent in pre-Euclidean geometry (see Heath 1921, 338-9 and 381-2; Mueller 1981, 187).

69 See McKirahan 1992, 157-8; Ebert and Nortmann 2007, 749-50.

70 Specifically, they can be viewed as instances of the schema described in n. 41 above.

71 See Philoponus, *in APr.* xiii.2, 254.12-23 Wallies; cf. n. 67 above.

The major premisses that are missing in A7 and A8 are theorems of geometry. The one missing in A9 is an axiom (that is, an unproved principle common to more than one science):[72] it is one of the common notions listed by Euclid at the beginning of the *Elements*. Thus, the three missing premisses are true of necessity. Accordingly, the conclusions of A7-9 follow necessarily from the remaining premiss.[73] Yet, for Aristotle, A7-9 are not deductions just as A3 is not a deduction. If someone puts forward, in writing or speech, an argument such as A9, she has failed to produce a deduction. For example, consider the following argument put forward by Euclid in his proof that the base angles of an isosceles triangle are equal (*Elements* 1.5, 32-5):

> Since the whole angle ABG was proved equal to the angle ACF, and in these the angle CBG is equal to the angle BCF, the remaining angle ABC is equal to the remaining angle ACB.

This proof is parallel to A9. The first two clauses in Euclid's proof correspond to the premiss of A9, the last clause corresponds to the conclusion. Euclid does not mention a universal premiss to the effect that all remainders of subtracting equals from equals are equal. Of course, he mentioned it earlier as one of his common notions and is tacitly relying on it here. Nevertheless, he does not express it in writing in the passage just quoted. Thus, Aristotle will insist that Euclid has failed to produce a deduction. Since proofs are a kind of deduction, Euclid has failed to give a proof of his theorem.

A similar criticism of Euclid is articulated by Gottlob Frege in his essay 'On the scientific justification of a Begriffsschrift' (Frege 1964, 157):

> Even such a conscientious and rigorous writer as Euclid often makes tacit use of premisses to be found neither among his axioms and postulates nor among the premisses of the particular theorem being proved. For instance, in proving the 19th theorem of the first book of the *Elements* . . . , he makes tacit use of the statements:

72 *Post. An.* 1.2, 72a14-18; 1.10, 76a37-41, 76b20-1; 1.11, 77a29-31. See Barnes 1994, 99-100 and 138-9.

73 If a premiss that is true of necessity is taken away from an argument in which the conclusion follows necessarily from the premisses, the conclusion still follows necessarily from the remaining premiss(es).

(1) If a line is not larger than another line, then it is equal to the other or smaller than it.
(2) If an angle is equal to another angle, then it is not larger than the other.
(3) If an angle is smaller than another angle, then it is not larger than the other.

Frege is referring to passages such as the following (Euclid, *Elements* 1.19.13-15):

> AC is not smaller than AB. And it was proved that it is not equal either. Therefore AC is larger than AB.

Euclid's argument in this passage tacitly relies on a missing premiss, namely Frege's (1). Thus, the argument is deficient in much the same way as the earlier argument corresponding to A9.[74] In both Aristotle's and Frege's view, some of Euclid's arguments are deficient because he fails to make explicit all the premisses on which they rely. In order to detect and avoid such deficiencies, Frege introduces his Begriffsschrift (1879, p. iv):

> It was important to keep the chain of inferences free of gaps. In attempting to comply with this requirement in the strictest possible way I found the inadequacy of language to be an obstacle ... This deficiency led me to the idea of the present Begriffsschrift. Its first purpose, therefore, is to provide us with the most reliable test of the validity of a chain of inferences and to point out every assumption that tries to sneak in unnoticed, so that its origin can be investigated.[75]

Frege's Begriffsschrift is primarily a tool for detecting gaps, i.e. missing premisses, in deductive arguments. For example, if Euclid's argument quoted above is translated into Begriffsschrift, it becomes clear that a premiss is missing in it.[76] Similarly, Aristotle uses his syllogistic as a tool for detecting missing premisses. When Aristotle denies in *Prior Analytics* 1.24 that arguments such as A7-9 are deductions even though their conclusion follows necessarily from the premiss, he does so on the basis of his syllogistic; for none of these arguments

74 Except that in the present argument, the missing premiss is not mentioned by Euclid as one of his common notions.

75 Similarly, Frege 1884, §91; 1893, pp. vi-vii.

76 See Weiner 1990, 52-3 and 67; 2010, 43-4.

contains a universal premiss, whereas the syllogistic requires that every deduction contain a universal premiss. Thus James Allen writes (2001, 24):

> Aristotle conceived the categorical syllogistic as, among other things, a way of bringing out and making explicit the often unstated premisses because of which the conclusion of a syllogism follows of necessity . . . The analysis of arguments with the aid of the categorical syllogistic uncovers assumptions on which they depend that often go unnoticed and unsaid.

Aristotle's syllogistic provides a criterion for determining whether or not premisses are missing in a given deductive argument. As we will see in the next section, this is something that distinguishes the *Prior Analytics* from the *Topics*.

5 A Criterion for Gapless Deductions

While the issue of missing premisses is prominent in the *Prior Analytics*, it is largely absent from the *Topics*. In the eighth book of the *Topics*, Aristotle occasionally refers to 'necessary premisses' (ἀναγκαῖαι προτάσεις), by which he means premisses that are necessary to deduce the conclusion in an argument.[77] If not all of them are present, a premiss is missing. However, Aristotle does not, in the *Topics*, undertake to determine which premisses are the necessary ones in a given argument. Nor does he criticize arguments just because they do not contain all the necessary premisses. He only criticizes arguments if they omit those necessary premisses that are less reputable than the conclusion or inferior to the premisses which are present in the argument (*Topics* 8.11, 161b26-8):

> The third criticism of an argument applies if a deduction does come about (γίνοιτο) with certain premisses added, but these are inferior (χείρω) to the ones asked for and less reputable (ἔνδοξα) than the conclusion.

What Aristotle does not say in this passage, or elsewhere in the *Topics*, is that arguments are open to criticism if one of their necessary premisses is missing. The implication is that it is acceptable to omit necessary premisses as

77 Aristotle characterizes necessary premisses as those 'through which the deduction comes about' (δι' ὧν ὁ συλλογισμὸς γίνεται, *Top.* 8.1, 155b19-20 and 29). He mentions them a few times later on (e.g. 8.1, 155b36, 156a10, 157a12; 8.11, 161b29-30), but does not discuss them in any detail.

long as they are more reputable than the conclusion and superior to the other premisses.[78] This does not mean that arguments in which such premisses are omitted count as deductions. On the contrary, Aristotle's use of the verb 'come about' (γίνοιτο) in the passage just quoted suggests that they are not deductions. Crucially, however, Aristotle does not feel the need to criticize such arguments. For the purposes of the *Topics*, they are perfectly acceptable. For example, if the major premisses of A4-6 are more reputable than the conclusion and superior to the minor premisses, they may be omitted. Thus, A7-9 will be acceptable arguments even though they are not deductions.

This may be part of the reason why, in the *Topics*, Aristotle does not attempt to determine the set of premisses that are necessary for a given argument to count as a deduction. More important, it is doubtful whether he would be in a position to do so in the *Topics*. There is no single, obvious way to determine the set of these premisses. For example, someone might maintain that, as soon as the conclusion of an argument follows necessarily from the premiss(es), the argument counts as a deduction and no further premiss is needed. On such an account, contrary to Aristotle's view, A3 and A7-9 qualify as deductions.

Conversely, someone might contend that premisses are missing even in arguments which Aristotle regards as deductions. For example, consider the following straightforward argument in Barbara:

A10 All animals are mortal.
All humans are animals.
Therefore, all humans are mortal.

For Aristotle, this is a deduction. No premiss is missing. However, some ancient theorists denied that A10 is a deduction and insisted that a premiss is missing in it, namely a premiss such as 'If all animals are mortal and all humans are animal, then all humans are mortal'. We know that this was the view of Stoic theorists whom Alexander calls 'moderns' (νεώτεροι).[79]

78 Alexander, *in Top.* ii.2, 568.13-18 Wallies. Similarly, Aristotle states that arguments are subject to criticism if 'the conclusion does not come about either when some premisses are taken away or some premisses are added' (*Top.* 8.11, 161b22-4). Again, this implies that an argument in which premisses are missing is not subject to criticism as long as it can be turned into a deduction by adding suitable premisses; see Allen 1995, 189; Rapp 2000, 27-32.

79 Alexander, *in APr.* ii.1, 262.28-9, 345.13-18, 390.16-18 Wallies; see Mueller 1969, 179-80; Frede 1974*a*, 2-5 and 10; Barnes 1990, 73-5 and 114-16.

Alternatively, one might point out that Aristotle takes the validity of Barbara to depend on the *dictum de omni* (that is, his explanation of the meaning of phrases such as 'is predicated of all').[80] Accordingly, the validity of A10 depends on a version of the *dictum de omni* explaining the meaning of the construction 'All . . . are ___'. Based on this, one might argue that A10 lacks a premiss, and that a suitable version of the *dictum de omni* needs to be added as an extra premiss in order for this argument to count as a deduction.

Or, one might argue that A10 lacks a premiss to the effect that the middle term, 'animals', has the same meaning in both premisses. After all, if the middle term were ambiguous and did not have the same meaning in the two premisses, then A10 would fail to be a deduction. For example, consider the following fallacious argument:

A11 All circles are geometrical figures. (TRUE)
Homer's poem is a circle. (TRUE)
Therefore, Homer's poem is a geometrical figure. (FALSE)

The premisses of this argument are understood in such a way that they are both true; the conclusion is false. As Aristotle sees it, this is because the middle term, 'circle', does not have the same meaning in the two premisses. As a result, A11 is not a deduction but a fallacy (παραλογισμός).[81] At the same time, he states in the *Sophistici Elenchi* that A11 can be turned into a deduction by adding a premiss to the effect that the middle term has the same meaning in both premisses.[82] Even though this premiss is false, it would turn A11 into a valid deductive argument. Accordingly, if the middle term of an argument in Barbara such as A10 did not have the same meaning in both premisses, the argument would fail to be a deduction just as A11. Thus, since the validity of arguments such as A10 depends on the assumption that the middle term is not used ambiguously, one might argue that in order for such arguments to count as deductions this assumption needs to be made explicit as an additional premiss.

Of course, Aristotle would reject such a view. Still, it is not obvious how he would justify rejecting it. The above considerations illustrate the difficulty of determining, in a principled way, whether or not a premiss is missing in a given argument. Aristotle's definition of deduction in itself does not provide

80 *APr.* 1.4, 25b37-40; see n. 49 above.

81 *APo*. 1.12, 77b27-33, *SE* 10, 171a5-11; see Pacius 1597*a*, 810; Poste 1866, 124; von Kirchmann 1883, 21; Barnes 1994, 149-50; Dorion 1995, 275.

82 *SE* 8, 170a12-19; 22, 178a24-8; see Fait 2007, pp. xxii-xxv; 2013, 243-50.

a criterion by which this question can be decided. In the *Topics*, Aristotle does not supply such a criterion.[83] The syllogistic in the *Prior Analytics*, however, provides one. By means of the syllogistic Aristotle is in a position to assert, for example, that no premiss is missing in A10, but that premisses are missing in A3 and A7-9.[84] This is not to say that he would be in a good position ultimately to explain *why* no premiss is missing in A10 and *why* the moderns are wrong in thinking otherwise. But, unlike in the *Topics*, he can now justifiably assert *that* no premiss is missing. Thus, as Burnyeat (1994, 15) points out, Aristotle introduces the syllogistic . . .

> not as a further contribution to the definition of *sullogismos*, which the *Prior Analytics* repeats from the *Topics*, but as a way of testing when an argument is valid in the sense thereby defined and when it is not.

Aristotle holds that every deduction 'comes about through the three figures'.[85] By this he means that every deduction conforms to, or contains a part that conforms to, the valid schemata in the three syllogistic figures. He acknowledges a class of what he calls deductions from a hypothesis (συλλογισμοὶ ἐξ ὑποθέσεως), which cannot be completely analyzed as instances of these schemata (although they contain a part that can be so analyzed).[86] It is disputed whether Aristotle took these deductions from a hypothesis to be genuine deductions satisfying his official definition of deduction.[87] If he did, then his test is not complete in that it does not fully capture all deductions. But whether or not it is complete, the test is clearly sound with respect to his definition of deduction: whenever an argument instantiates one of the schemata in the three figures (or a series of these schemata), then no premiss is missing and the argument satisfies his definition of deduction.

83 See Allen 1995, 189-90; 2001, 53 and 69.

84 Moreover, Aristotle is in a position to determine the precise number of premises that should be present in a deduction given the number of terms occurring in it: in every deduction, he argues, the number of premisses is one less than the number of terms (*APr.* 1.25, 42b1-16). In the *Topics*, by contrast, Aristotle is not able to specify the number of premises that should be present in a deduction.

85 *APr.* 1.23, 40b20-2; 41b1-5; 1.25, 42a30-1; 1.28, 44b6-8; 1.29, 45b36-46a2; 2.23, 68b9-13. See Allen 2001, 21; Primavesi 1996, 60.

86 *APr.* 1.23, 41a21-b5; 1.44, 50a25-32 and 50b2-3. See Lear 1980, 34-5 and 39-41; Striker 1998, 210-17; Speca 2001, 13-14.

87 Some commentators deny that he took them to be genuine deductions (Barnes 1997, 164-6; Bobzien 2002, 371 n. 32), whereas others affirm it (Lear 1980, 41-2; Slomkowski 1997, 128-9; Crivelli 2011, 179-81).

By contrast, Aristotle's test is not sound with respect to the Stoic conception of deduction (see n. 79 above). For the Stoics, none of the arguments endorsed by Aristotle in his syllogistic counts as a genuine deduction. On the other hand, Aristotle's syllogistic is (largely) sound with respect to contemporary conceptions of what counts as a valid deductive inference. While some parts of it might be affected by what is known as the problem of existential import, the rest of Aristotle's syllogistic is sound from the perspective of modern formal logic.[88]

Given the soundness of his syllogistic, Aristotle can maintain 'that syllogistic is a universal test of formal or deductive validity' (Burnyeat 1982, 201), in much the same way that Frege conceived of the Begriffsschrift as 'the most reliable test of the validity of a chain of inferences' (Frege 1879, p. iv).

6 Eliminating Ambiguous Terms

According to Aristotle's syllogistic, no premiss is missing in instances of Barbara such as A10. In particular, there is no need for an additional premiss to the effect that the middle term is not used ambiguously. In general, questions of ambiguity are not discussed in the *Prior Analytics*. On the other hand, they are prominent in the *Topics* and *Sophistici Elenchi*. Homonymy is one of the thirteen sources of fallacy identified in the *Sophistici Elenchi* (*SE* 4, 165b23-166a6). In the *Topics*, the ability to detect ambiguities is listed as one of the four 'tools by means of which we may be well equipped with deductions' (*Top.* 1.13, 105a21-5; see also *Top.* 1.15). The tool is useful, among other things, for resisting fallacies such as A11 (*Top.* 1.18, 108a26-9). In other words, it is useful for determining whether or not a given argument is a genuine deduction. Given this, it is noticeable how little attention Aristotle pays to the issue of ambiguity in the *Prior Analytics*. When he asserts the validity of schemata such as Barbara

88 In the *Begriffsschrift* (1879, §12), Frege explains how to translate Aristotle's four main kinds of assertoric sentences into his own quantifier logic. On this translation, all schemata endorsed by Aristotle in the assertoric syllogistic are valid in modern quantifier logic, except those that are affected by the problem of existential import (see e.g. Beaney 1996, 49-56). Accordingly, it is sometimes thought that Aristotle's 'assertoric syllogistic is not purely formal' because it relies on tacit assumptions of existential import (Rini 2011, 28; similarly, Oliver and Smiley 2013, 182-3). On the other hand, there are a number of ways to translate Aristotle's assertoric syllogistic into modern quantifier logic in such a way that the problem of existential import does not arise (see e.g. Malink 2013, 41-4 and 66-8). On such a translation, the entire assertoric syllogistic is sound with respect to modern quantifier logic.

and Darii, he does not warn the reader that they may fail if the terms involved are ambiguous. Still, he must be aware that ambiguity poses a threat to the validity of his schemata. Presumably he thinks that fallacious arguments due to ambiguity do not qualify as instances of his valid schemata. For example, the following argument must not qualify as an instance of Darii:

A12 All circles are geometrical figures. (TRUE)
Some poem is a circle. (TRUE)
Therefore, some poem is a geometrical figure. (FALSE)

If this argument were an instance of Darii, then Darii would be invalid (since any schema that has an instance with true premisses and a false conclusion is invalid). Thus, Aristotle ought to be able to exclude from his syllogistic troublesome ambiguities such as the one in A12. He must ensure that the terms for which his schematic letters are placeholders are not ambiguous.[89] Arguably, Aristotle is in a good position to ensure this without resorting to arbitrary stipulation. This can be seen as follows. Susanne Bobzien has shown that, according to the *De Interpretatione*, a declarative sentence that contains an ambiguous term is not one but more than one affirmation or denial (*Int.* 8, 18a18-26), just as a question sentence that contains an ambiguous term is not one but more than one question (*SE* 17, 175b39-176a18).[90] At the same time, Aristotle requires in the *De Interpretatione* that every genuine affirmation and denial be one and not more than one affirmation and denial (that is, a sentence in which one thing is affirmed or denied of one thing).[91] Consequently, a sentence that contains an ambiguous term does not count as a genuine affirmation or denial. Now, in the first chapter of the *Prior Analytics*, the premisses of deductions are defined as genuine affirmations and denials.[92] The same is

89 Thus, Atherton (1993, 409) holds that Aristotle's syllogistic 'has no proper place for ambiguity'. She argues that, in the *Prior Analytics*, 'Aristotle sees himself as working with words and their complexes ... but seems to have assumed total elimination of ambiguities at the level of the logical "object-language"' (Atherton 1993, 458). Likewise, Harris (2009, 99) argues that, in the *Prior Analytics*, 'the elimination of homonymy has to be a practical proposal if the syllogism has to stand on its own two feet. Nowhere does Aristotle attempt to face up to this. Instead, he tacitly assumes that the requirement can somehow be met.' As I will argue shortly, Aristotle is in a better position than Harris suggests to justify the elimination of homonymous terms from the syllogistic.

90 Bobzien 2005, 258-64; 2007, 301-12.

91 *Int.* 8, 18a12-26; 10 19b5-7; 11 20b12-26.

92 *APr.* 1.1, 24a16-17 (see the beginning of Section 2 above). As a result, every premise of a deduction is required to be one and not more than one affirmation, denial, or question.

true for the conclusions of deductions. Thus, the premisses and conclusions of deductions cannot contain ambiguous terms; for otherwise they would not be genuine, unitary affirmations and denials.

This also helps explain why Aristotle did not deem it necessary to discuss ambiguity or homonymy in the *Prior Analytics*. By characterizing the premisses of deductions as genuine affirmations and denials in the first chapter of the *Prior Analytics* he makes it clear that ambiguous terms have no place in the syllogistic.[93] Of course, Aristotle realizes that ambiguous terms abound in ordinary language. But, as he explains in *Metaphysics* Γ.4, ambiguous terms can be eliminated from a language by introducing a new word for each of the various things signified by them.[94] The result is a language that does not contain any ambiguous terms. If I am correct, Aristotle's syllogistic is intended to apply to such a language.[95]

More generally, Aristotle's syllogistic seems to operate on the assumption that the sentences that serve as premisses and conclusions in deductions are not ambiguous.[96] This, I suggest, is another respect in which the syllogistic is

This is confirmed by *SE* 6, 169a6-18, where Aristotle regards this latter requirement as part of the definition of deduction (Malink 2014, 157-8). On this account, it follows from Aristotle's definition of deduction that the premisses of a deduction cannot contain ambiguous terms.

93 By contrast, the terminology of affirmation and denial is absent from the first book of the *Topics* and the characterization of premisses and problems in *Topics* 1.10-11. Unlike the *Prior Analytics*, the *Topics* does not seem to rely on the discussion of affirmations and denials in the *De Interpretatione*. This accords with the view that the *De Interpretatione* was written after the *Topics* but before the *Prior Analytics* (Bocheński 1956, 49-51; similarly, Sainati 1968, 203-4).

94 *Metaph*. Γ.4, 1006a34-b11; see Alexander, *in Metaph*. i, 277.9-14 and 277.20-31 Hayduck.

95 This does not prevent Aristotle in the *Prior Analytics* from using terms which are actually ambiguous. For example, he often uses 'good' and 'white' as examples of syllogistic terms, even though he thinks that they are ambiguous (*Top*. 1.15, 106a23-32, 106b4-12, 107a3-13, 107a36-b5, 107b13-18). This is harmless as long as he does not use them ambiguously. Aristotle may be assuming that, whenever such terms occur in the syllogistic, they are not ambiguous but signify only one of the things they ordinarily signify.

96 It is sometimes thought that indeterminate sentences are ambiguous (e.g. Jones 2010, 42-4). Since these sentences are admissible in the syllogistic, Keyt infers that 'Aristotle seems tempted to apply logic to ambiguous statements' (Keyt 2009, 33). However, it is not clear whether Aristotle took indeterminate sentences to be ambiguous in the syllogistic. He does not use the terminology of ambiguity in connection with indeterminate sentences. It is open to him to treat them as having a unitary meaning such that they are true just in case either the corresponding particular sentence or the corresponding universal sentence is true (i.e. just in case the corresponding particular sentence is true;

akin to modern systems of formal logic such as Frege's Begriffsschrift. In 'On the scientific justification of a Begriffsschrift', Frege writes (1964, 158):

> We need a system of symbols from which every ambiguity is banned, which has a strict logical form from which the content cannot escape.

Elsewhere (1979, 213; see also 227) he writes:[97]

> A sign must not be ambiguous. Freedom from ambiguity is the most important requirement for a system of signs which is to be used for scientific purposes.

The Begriffsschrift is designed to satisfy this requirement, and so are all systems of formal logic after Frege. Like Aristotle's syllogistic, they are formulated in languages that do not contain any ambiguous expressions.

7 Conclusion

We are now in a position to return to the question as to why the *Prior Analytics*, but not the *Topics*, qualifies as a treatise of formal logic. We have identified four distinctive features of the *Prior Analytics* that can help answer this question. First, the *Prior Analytics* abstracts from speaker meaning and only takes into account the literal meaning of the sentences involved in a deduction. Nothing of relevance is left to tacit understanding between speaker and hearer. Every aspect of the meaning that is relevant to an argument's counting as a deduction is made explicit by *some* linguistic expression, even if Aristotle is not formalistic and does not prescribe *which* expressions to use (Sections 2 and 3). Secondly, Aristotle is concerned to make explicit all premises that are necessary to deduce the conclusion in a given argument (Section 4). Thirdly, Aristotle provides a criterion for determining when all the necessary premises

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see n. 37 above). In the modal syllogistic, however, Aristotle does use ambiguous sentences. His expression for possibility (ἐνδέχεσθαι) is ambiguous between two kinds of possibility which are known as one-sided and two-sided possibility (cf. πολλαχῶς λέγεται τὸ ἐνδέχεσθαι at *APr.* 1.3, 25a37-8; ὁμωνύμως ἐνδέχεσθαι λέγομεν at 1.13, 32a20-1). Aristotle usually leaves it to the reader to determine which of the two he means in a given context. This is another respect in which the modal syllogistic falls short of the standards of precision set in the assertoric syllogistic (cf. n. 51 above).

97 See Weiner 2004, 102; cf. 2010, 43-4.

have been made explicit. The criterion is (largely) sound with respect to modern conceptions of valid deductive inference (Section 5). Fourthly, deductions are formulated in a language that is supposed to be free from homonymy and ambiguity (Section 6).

These four features are characteristic of the *Prior Analytics* but absent from the *Topics*. At the same time, they are emphasized by Gottlob Frege in his discussion of the Begriffsschrift and have been central to systems of formal logic ever since Frege. They are essential features of the discipline we today call formal logic. It is in virtue of these four features, I submit, that the *Prior Analytics* but not the *Topics* is a treatise of formal logic.

Three points of elaboration and clarification are in order. First, the third feature listed above includes the condition that Aristotle's criterion is (largely) sound with respect to modern conceptions of valid deductive inference (see n. 88 above). This condition is needed to preclude deficient criteria that fall short of widely accepted standards for formal logic. For example, someone might adopt the criterion that an argument counts as a deduction as soon as the conclusion follows necessarily from the premiss(es) (see Section 5). On such an account, a large number of arguments that are generally not regarded as formally valid would count as deductions (e.g. A3 and A7-9). Had Aristotle adopted such a criterion, the *Prior Analytics* would not qualify as formal logic. Hence, the four features should contain a condition that excludes this and other deficient criteria.[98]

The second point concerns the unity of the four features listed above. While the first three features can be grouped under the heading of 'making everything explicit', the fourth one seems to stand apart. A case can be made, however, that this last feature falls under this heading as well. For if ambiguous terms were admitted, the validity of deductions in Barbara would require an additional premiss to the effect that the two occurrences of the middle term

98 One might suggest that this condition should require soundness, not with respect to modern conceptions of valid deductive inference, but with respect to Aristotle's definition of deduction. In this case, however, the condition would not be true of modern systems of formal logic, since Aristotle rejects a large number of inferences validated by these systems (e.g. inferences that contain superfluous premisses or whose conclusion is among the premisses; see nn. 59 and 67 above). Consequently, the four features would fail to account for the intuition that there is a single discipline of formal logic that was founded by Aristotle and persists to the present day (which is implicit in the generally accepted view that Aristotle was the founder of formal logic). Thus, it seems preferable to include a condition requiring soundness with respect to modern conceptions of valid deductive inference.

have the same meaning.[99] In order for this additional premiss to say what it is supposed to say, it would need to contain an expression that picks out one or both occurrences of the middle term. But if terms are allowed to be ambiguous, there is no guarantee that this expression is not ambiguous and that it has the right meaning to pick out the two occurrences of the middle term. Yet another premiss will be needed to connect the additional premiss to the two original premisses by stating that the expression in question picks out the two occurrences of the middle term—and so on to infinity. Thus, acceptance of ambiguous terms leads to an infinite regress of premisses. As a result, the project of making explicit all the premisses on which a given deduction depends becomes impossible. Aristotle's decision to exclude ambiguous terms in the syllogistic is a way to block this regress. Thus, the fourth feature listed above contributes to the goal of making explicit all the premisses that are necessary in order for an argument to count as a deduction, by ensuring that the goal is feasible in the first place. If this is correct, the four features are unified in that they all contribute to the same goal.

Finally, one might wonder whether the four features suffice to account for the syllogistic's status as formal logic. There are a number of other features that might be thought to contribute to this status. For example, the syllogistic is reductive in that the validity of second- and third-figure schemata is derived in a systematic manner from that of first-figure schemata by means of conversion rules and proof by *reductio*. Gisela Striker argues that the presence of such reductive derivations is 'one of the most important differences between the *Prior Analytics* and the *Topics*' (1996, 204). By contrast, the four features specified above do not require such derivations, but are compatible with a mere collection of valid schemata lacking any reductive structure. In my view, this is acceptable. To see why, compare Aristotle's syllogistic to his account of demonstrative science in the *Posterior Analytics*. Although Aristotle does not seem to regard the syllogistic as a science in the strict sense, its structure closely resembles that of a demonstrative science: the first-figure schemata and conversion rules correspond to the unproved principles of the science, the second- and third-figure schemata correspond to the proved theorems, and Aristotle's derivations of the latter from the former in *Prior Analytics* 1.5-6 correspond to the demonstrations in the science.[100] Had Aristotle omitted the derivations, the syllogistic would lack demonstrative structure. But this does not mean that it would not count as a treatise of formal logic. In the development of a science,

99 See n. 82 above, and the discussion of A11 in Section 5. Similar premisses will be needed for the major and minor terms.

100 See Barnes 2007, 362-9.

demonstration and explanation is typically preceded by a preliminary inquiry collecting the relevant facts (*Posterior Analytics* 2.1, 89b23-31). For example, the *Historia Animalium* presents a collection of zoological facts largely without offering scientific demonstrations and explanations of them, leaving the latter task to treatises such as the *De Partibus Animalium*.[101] But the *Topics* does not stand to the *Prior Analytics* as the *Historia Animalium* stands to the *De Partibus Animalium*; the *Topics* does not contain a collection of second- and third-figure schemata for which the *Prior Analytics* supplies the appropriate derivations by reducing them to first-figure schemata. Instead, I suggest, the crucial difference between the *Topics* and the *Prior Analytics* is captured by the four features identified above: just as the *Historia Animalium* counts as a zoological treatise even if it does not contain the completed demonstrative science of zoology, any treatise which possesses the four features will qualify as a treatise of formal logic even if it lacks the demonstrative structure of a reductive system.[102] If this is correct, the four features suffice to explain why the *Prior Analytics* is a treatise of formal logic. Had they been present in the *Topics*, then the *Topics* would mark the beginning of formal logic.

The four features are aimed at making fully explicit all premisses that are necessary for a given argument to count as a deduction. By setting up this aim, Aristotle has established one of the central tenets of formal logic. I.M. Bocheński (1948, 52) writes:

> The whole business of Formal Logic is to make tacit assumptions of reasoning explicit.[103] To deny the thesis [that in Formal Logic all tacit assumptions must be made explicit] would, consequently, be to contradict the very nature of Formal Logic, not only as it is now, but as it has always been since the time of Aristotle.

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101 See e.g. Lennox 2001, 123-4; Leunissen 2010, 79-80.

102 Aristotle's reductive derivations in *Prior Analytics* 1.5-6 are proofs that satisfy the four features. If they did not satisfy these features, they would presumably not help explain the syllogistic's status as formal logic. Again, this suggests that the presence of reductive derivations in itself does not contribute to this status, but only the fact that these derivations conform to the standards described by the four features.

103 Similarly, John MacFarlane (*forthcoming*, Section 1) describes a view of logic on which 'the role of logic is to help us make explicit everything on which an inference depends. When we have teased out hidden assumptions to the point where we have a formally valid argument, then we know that the process of explicitation has come to an end; we have made all of the assumptions on which the inference depends explicit'.

I hope to have shown that Bocheński is right. While the specific account of deduction provided by Aristotle's syllogistic may play only a minor role in contemporary logic, his *Prior Analytics* initiated a project which continues to define the discipline of formal logic today.[104]

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104 Earlier versions of this paper were presented in Charlottesville, Chicago, Geneva, Groningen, Munich, New York, Oxford, Paris and Vancouver. I would like to thank the audiences for very helpful comments and discussion, especially Simona Aimar, Jonathan Barnes, Susanne Bobzien, Sarah Broadie, David Bronstein, Laura Castelli, Alan Code, Ursula Coope, Adam Crager, Paolo Crivelli, Daniel Devereux, Paolo Fait, Pieter Hasper, Brad Inwood, Gabriel Lear, Jonathan Lear, Harvey Lederman, Mitchell Miller, Ben Morison, Jessica Moss, Oliver Primavesi, Christof Rapp, Jacob Rosen, Paul Thom and an anonymous reader for *Phronesis.*

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